Econ 802

First Midterm Exam

Greg Dow October 11, 2013

All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Let $y_1 \leq 0$ be an input and $y_2 \geq 0$ be an output. The production plan $y = (y_1, y_2)$ is in the feasible set Y if <u>either</u> $-1 < y_1 \leq 0$ and $y_2 = 0$; <u>or</u> $y_1 \leq -1$ and $0 \leq y_2 \leq 1 - y_1$.

(a) Draw a graph of Y. Is this set closed? Bounded? Convex? Explain briefly.

(b) Consider price vectors $p = (p_1, p_2) > 0$. For what set of price vectors is the profit function $\pi(p)$ well defined? Use a graph to justify your answer.

(c) In period one the firm faces the prices $p_1 = 3$ and $p_2 = 1$. It chooses $y = (0, 0)$. In period two the firm faces the prices $p_1 = 3$ and $p_2 = 2$. It chooses $y = (-1, 2)$. Use a graph to show the <u>smallest</u> convex monotonic production set YI consistent with these choices. On another graph, show the <u>largest</u> convex monotonic production set YO consistent with these choices. Then explain why either YI or YO would be consistent with the firm's observed choices.

2. A firm has one output $y \geq 0$ and two inputs $x = (x_1, x_2) \geq 0$. All prices (p, w_1, w_2) are positive. Using each of the methods listed below, show that when the output price p rises, holding the input prices constant, the firm does not produce any less output and it may produce more. State any assumptions you need in order to use each method. (<u>Note</u>: if a matrix needs to be inverted, you don't have to carry out the calculation, just say what the results would be.)

(a) Use the weak axiom of profit maximization.

(b) Apply Hotelling's Lemma to the profit function.

(c) Differentiate the first order conditions for profit maximization.

3. The cost function $c(w, y)$ is differentiable with $w = (w_1 \,.\,.\, w_n) > 0$ and $y > 0$.

(a) Prove that $c(w, y)$ is homogeneous of degree one in the price vector w.

(b) State Shephard's Lemma and prove that it is true.

(c) Let $\partial c(w, y)/\partial w$ be the vector of derivatives of $c(w, y)$ with respect to the n input prices. Show that the scalar product of this vector with the price vector w is equal to $c(w, y)$. Briefly explain your reasoning.

4. Consider the production function $f(x) = \min \{ax_1 , bx_2\}$ where $a > 0$ and $b > 0$. The input prices are $w = (w_1, w_2) > 0$.

(a) For a fixed output y, use a graph to minimize cost wx subject to $y = f(x)$ and then derive the long run cost function $c(w, y)$. Briefly explain your reasoning.

(b) Now assume input 2 is fixed at $x_2^* > 0$ in the short run. Derive the short run cost function $c(w, y, x_2^*)$ and state the range of output levels for which this function is well defined. Then draw a graph showing average fixed cost, average variable cost, marginal cost, and average total cost. Briefly explain your reasoning.

(c) Let $x_2(y)$ be the optimal level of input 2 in the long run. Fix an output level $y^* > 0$ and define $x_2^* = x_2(y^*)$. Draw a graph showing long run average cost $c(w,y)/y$ and short run average cost $c(w, y, x_2^*)/y$ as functions of y. Be accurate about the shapes of these curves and the range of output levels for which short run average cost is well-defined. Briefly explain your reasoning.

5. Here are a few miscellaneous questions. Your answers don't need to be long.

(a) The production function is $f(x) = (x_1x_2)^a$ where $a > 0$. Compute the elasticity of substitution σ. What does this tell you about the fraction of total cost spent on each input? Explain briefly.

(b) The production function is homogeneous of degree k so $f(tx) = t^k f(x)$ for every $t > 0$. Compute the local elasticity with respect to scale $e(x)$. What does this tell you about the shape of the long run average cost curve? Explain briefly.

(c) You are solving a profit maximization problem with the production function $f(x)$. Explain in words why it would be helpful to assume each of the following global conditions: (i) $f(x)$ is concave; (ii) $f(x)$ is strictly concave; (iii) the Hessian matrix of $f(x)$ is negative definite everywhere.